The runtime effici			o grapi	. argorn			
. Background							
BFS							
. DFS . Dijkstra							
. Background							
At start: a digraph $G = (V)$	/,E) possibl	y with	weights o	n edges			
Q : What is the size of	f a digrap	h?					
A: We need to use tw						_	
						_	
n =	the numb	er of	nodes	m = th	e number	of edges	
We assume: at most of	one edge f	rom x	to y.				
	2 75 .	11			2		
For a digraph: $m \leq n$	r. Typica	Hy, m	is much	less than	n². —		
Adjacency sum resul	t						
Let the nodes of a		e 1, 2	$, 3 \ldots n.$	Let the n	umber of	adjacent i	nodes to
node i be r_i . Then				MID OVERSEL			
_		$r_1 + r$	2 + +	$r_n = m$			
We will consider uppe	r bound or	n effic	iency: res	ults will b	oe O()		

2. BFS							
1 BI	READTH-FIRST-SEA	ARCH(s C)					
	xecutes a breadt		arch on g	raph G st	tarting	from	
	ource node s		8	2 10 10		_	
4							
5 f c	orEach node x in	G				_	
6	x.color = white,	$x.d = \infty$, $x.\pi$	r = NIL				
	nd					_	
8		141	er o	0.00			
	* Give source no		iate valu	es. */		_	
	$color = { m gray} , s.d \ { m initialize} { m a} { m q}$						
	NQUEUE (Q,s)	ueue III Q				_	
	hile Q is not en	mpty					
14	x = DEQUEUE(Q)	проз				_	
15							
— 16	forEach node y	in $x.Adj$				_	
17	if $y.color ==$						
18		y is undisco				_	
19		$\operatorname{gray}, \ y.d = x$	$d+1$, $y.\pi$	= x			
20	ENQUEUE($Q_{\cdot},y_{\cdot})$				_	
21 22	end end						
$\frac{22}{23}$	x.color = black					_	
	ad Diack						
THREE LOC)PS						
forEach (1	ines 5-7)						
	processed once.						
_ line 6: $O(n$).						

whi	le-loop	(line	13)											
Each	node i	s adde	d to	Q at	mos	t onc	e. W	hy?						
	Node	will be	adde	ed to	Q o	nly w	hen i	it is v	vhite	(line	e 17)			
	When													
•	Inside	the w	hile-	loop,	, a no	ode's	color	is ne	ever s	set to	whi	te.		
Cons	sequenc	e: the	whil	e-loo	op ha	s at 1	most	n-ite	ratio	ns.				
	line 14	1: O(n) (ass	sumir	ng Di	EQU.	EUE'	s effi	cicen	cy is	O(1)) —		
	line 23	3: O(n)											
	1975 76 Sing Sin													
Assur Conse	ach (linme each equence Ill node	node : owin	is add g to	adjao										
Assur Conse for a	me each equence	node: owines is m	is add g to 9: <i>O</i>	(m)	cency	sum	resu	lt, to	tal n	umbe	er of			
Assur Conse for a	me each equence Il node lines 17	node: owines is m	is add g to 9: <i>O</i>	(m)	cency	sum	resu	lt, to	tal n	umbe	er of			
Assur Conse for a	me each equence Il node lines 17	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			
Assur Conse for a	me each equence Il node lines 17 line 20:	node: owing six m and 1 $O(m)$	is add g to 9: O	(m)	g EN	sum	resu	lt, to	tal n	umbe	P(1))			

3. DFS	
J. DI J	
1	DEPTH-FIRST-SEARCH (s, G)
2	executes a depth first search on graph G starting from
3	source node s
4	
5	forEach node x in G
6	$x.color = white, x.\pi = NIL$
7	end
8	5 1 11 11 11 11 11 11 11 11 11 11 11 11
- 9 10	\triangleright initialize a stack in S
$\frac{10}{11}$	PUSH(S,s) while S is not empty
12	x = POP(S)
13	
14	/* If x is white, then it has not yet been discovered.*/
15	if $x.color ==$ white then
16	/* x is discovered. Put x back into the stack and
	investigate nodes adjacent to $x.*/$
17	$x.color = \operatorname{gray}, \operatorname{PUSH}(S, x)$
- 18	
19	for Each node y in $x.Adj$
- 20	
21	$PUSH(S,y), y.\pi = x$
- 22	else if $y.color == gray$ then
23 - 24	▷ a cycle that includes edge (x, y) exists end
25 25	end
_ 26	end
27	else
28	x.color = black
29	end
_ 30	end
TUDE	TLOOPS
INKE	E LOOPS
forEac	ch (lines 5-7)
	CONTROL OF THE PROPERTY OF THE
Laci ii	ode processed once: $O(n)$.

forEac	h (lin	es 16-	-22)													
This fo	rEach	-loop	is onl	y ever	run (once	for e	ach 1	iode	<i>x</i> . W	hy?					
• L	ines 17	-26 or	ıly exe	ecuted	whe	n x.ce	olor	is wł	nite a	t line	12.					
• A	t line 1	l7. a v	vhite:	node i	s colo	red g	grav.									
										1:4						
• 11	iside w	nne-i	.oop, a	i node	's col	or is	neve	r set	to w	mite.						
Concor	uonee:	Eon c	nah n-	do =	the f	nF-	ah 1-	on L	00.00	vo ites	ation	for -	nak -	odo -	disc	ant f
Conseq x . Owi				25												
	ne 20:										338 (371.6)			,		
• li	ne 21:	O(m)	(assu	ming	PUSF	I's ef	ficice	ency	is O	(1))						
• li	ne 23:	O(m)	(assu	ming	opera	tion	at lin	ne 23	are	O(1))					
while-	loop (l	ine 13	3)	I	1	I	I			1	l	I	I	I		
This lo	oop is ru	ın at n	nost m	+ 1 tin	mes. V	Vhy?										
1	ine 10	ie run	once a	nd line	91 is	run s	it mo	st m	times	. Her	ice at	most	$m \perp 1$	item	e	
_	re push				21 10	Tun c		150, 116	cilic	, 1101	ice an	mose	116	recm	13.5	
8				NG COOK												
)					ce .	1									
	At each	iteratio	on one	item is	s takei	ı off t	he st	ack (l	ine 1:	2).						
	At each							souther (S)		2).						
• <i>I</i>	At each	O(m)	(assum	ing PC				souther (S)		2).						
• <i>I</i>	At each	O(m)	(assum	ing PC				souther (S)		2).				ı		
• <i>I</i>	At each	O(m)	(assum	ing PC				souther (S)		2).						
• 1	At each ine 12:	O(m) (and 28	(assum	ing PC	OP's ef	ficicer	ncy is	O(1)))		ı) —					
• <i>I</i>	At each ine 12:	O(m) (and 28	(assum	ing PC	OP's ef	ficicer	ncy is	O(1)))		<i>i</i>) —					
• 1	At each ine 12:	O(m) (and 28	(assum	ing PC	OP's ef	ficicer	ncy is	O(1)))		n) —					
• 1	At each ine 12:	O(m) (and 28	(assum	ing PC	OP's ef	ficicer	ncy is	O(1)))		ı) —					

```
4. Dijkstra
        RELAX(x, y)
2
         if y.d > x.d + w((x,y)) then
3
            y.d = x.d + w((x, y)), y.\pi = x
4
        end
         DIJKSTRA(s, G)
  1
  2
         for Each node x in G
            x.color = white, x.d = \infty, x.\pi = NIL
  3
  4
         end
  5
  6
         s.color = gray, s.d = 0
  7
         \triangleright initialize a priority queue Q
  8
         INSERT(Q, s, 0)
  9
         while Q is not empty
            x = \text{EXTRACT-MIN}(Q)
 10
            /* Test whether shortest path to x has been found. */
 11
            if x.color \neq black then
 12
              forEach node y in x.Adj
 13
                  y.old = y.d, RELAX(x, y)
 14
 15
                  if y.color == white then
                      /* Node y is undiscovered. */
 16
 17
                      y.color = gray, y.\pi = x
                      INSERT(Q, y, y.d)
 18
                  else
 19
                     if y.d < y.old and y.color \neq black then
 20
                       /* Take into account that y.old < y.d. */
 21
 22
                       INSERT(Q, y, y.d)
 23
                    end
 24
                  end
              end
 25
              x.color = black
 26
 27
            end
 28
         end
```

RELAX has effic	eiency of $O(1)$						
THREE LOOP	<u>s</u>						
forEach (lin	es 5-7)						
Each node pr		ce: $O(n)$.					
forEach (line	s 16-28)						
• If <i>x</i> is no	t black at li	4, then the formula in 14, it is sentence a node is	t to black a	fter lines 16	–28 are e	xecuted.	
• If <i>x</i> is no	ack at line 1 of black at line 1 hile-loop, or For each note to the adjaces size of Q is	ine 14, it is sence a node is ode x the for le ency sum res	et to black a black, its co Each -loop l	fter lines 16 blor does no nas one itera	-28 are extended to the change.	each node	W
 If x is not Inside w Consequence: to x. Owing to iterations. Consequence: lines 17, 	ack at line 1 of black at line 1 hile-loop, or For each no to the adjactive of Q is 18 : $O(m)$	ine 14, it is sence a node is ode x the for le ency sum res	et to black a black, its co Each -loop b sult, the for	fter lines 16 blor does not has one itera •Each-loop	-28 are extended to the change. ation for the chas a tot	each node al of (at	W
 If x is not Inside w Consequence: to x. Owing to iterations. Consequence: lines 17, 	ack at line 1 of black at line 1 hile-loop, or For each no to the adjactive of Q is 18 : $O(m)$	ine 14, it is sentence a node is ode x the forlivency sum research at most m	et to black a black, its co Each -loop b sult, the for	fter lines 16 blor does not has one itera •Each-loop	-28 are extended to the change. ation for the chas a tot	each node al of (at	W
 If x is not Inside w Consequence: to x. Owing to iterations. Consequence: lines 17, 	ack at line 1 of black at line 1 hile-loop, or For each no to the adjactive of Q is 18 : $O(m)$	ine 14, it is sentence a node is ode x the forlivency sum research at most m	et to black a black, its co Each -loop b sult, the for	fter lines 16 blor does not has one itera •Each-loop	-28 are extended to the change. ation for the chas a tot	each node al of (at	W
 If x is not Inside w Consequence: to x. Owing to iterations. Consequence: lines 17, 	ack at line 1 of black at line 1 hile-loop, or For each no to the adjactive of Q is 18 : $O(m)$	ine 14, it is sentence a node is ode x the forlivency sum research at most m	et to black a black, its co Each -loop b sult, the for	fter lines 16 blor does not has one itera •Each-loop	-28 are extended to the change. ation for the chas a tot	each node al of (at	W

whi	le-lo	ор (1	ine 1	3)														
					m tii	mes.	Why?	•										
	An	item	is in	serted	Linto	prio	rity (only	v at l	ines	91 or	24	The	forE:	ach-le	oon h	asa	
						ation		ę om,	,	iiico		- 1.	1110	.012	ucii i	оор п	CLO CL	
									1	C	tha n				(1:no	1.4)		
•	Att	eacn 1	iterat	1011 01	ne ite	m is	такеп	remo	ovea	rom	tne p	riorit,	y que	ue Q	(line	14).		
•	line	14: (O(m l)	$\log_2 m$	i) (as	sumii	ng EX	TRA	CT-I	MIN i	s don	e wit	h hea	ap)				
	line	15: (O(m)															
Tot	al aff	icienc	v of	Diike	tra. ($\frac{1}{2}(n \perp$	m ±	$m \perp$	m loo	m J	- m lc	o m	-0	$(n \perp$	$m \log$	(m)		
100	ai cii	Сепс	y 01 .	DIJKS	ua.	/(16 +	111	116 +	nt log	2 111	The IC	82 111	<i>i</i> – 0	(11 +	11108	52 111)		
This	effic	iency	assu	mes 1	that th	ne pri	ority (queue	in D	jkstra	is im	plem	ented	usin	g a <u>he</u>	<u>eap</u> .		
								ted us	sing a	<u>bala</u>	nced	binar	y sea	rch tr	<u>ee</u> . T	he		
DIJKS	STRA	2 pro	cedur	e ass	umes	this.												

```
DIJKSTRA2(s, G)
1
 2
        forEach node x in G
           x.color = white, x.d = \infty, x.\pi = NIL
3
        end
4
5
 6
        s.color = gray, s.d = 0
7
        \triangleright initialize a priority queue Q
        INSERT(Q, s, 0)
 8
9
        while Q is not empty
10
           x = \text{EXTRACT-MIN}(Q)
           forEach node y in x.Adj
11
             y.old = y.d, RELAX(x, y)
12
               if y.color == white then
13
14
                  /* Node y is undiscovered. */
                  y.color = gray, y.\pi = x
15
                  INSERT(Q, y, y.d)
16
17
              else
                 if y.d < y.old and y.color \neq black then
18
19
                    /* Take into account that y.old < y.d. */
                   REMOVE(Q, y, y.old)
20
21
                   INSERT(Q, y, y.d)
22
                 end
23
            end
             x.color = black
24
25
           end
26
        end
Size of priority queue Q is (at most) n.
line 16 or 20 or 21: O(m \log_2 n)
```

Tämä teos on lis	ensoitu Creativ	e Commo	ns Nimeä-	EiKaupal	llinen-		
EiMuutoksia 4.0				tele lisen	ssiä osoit	teessa	
http://creativecor	Timoris.org/lice	115ES/DY-11	<u>C-110/4.0/</u> .				
tekijä: Frank Car	neron						
This work is licer						nny of	
NonCommercial- this license, visit						ру ог	
made by Frank (Cameron						
	~ \ (- \)						
(C)	≶ (≘)(
© (BY	NC ND						
© (BY	NC ND						
© ⊕ BY	NC ND						
© BY	NC ND						
© (BY	NC ND						
© ⊕ BY	NC ND						
© (I)	NC ND						
© ⊕ BY	NC ND						
ВУ	NC ND						
ВУ	NC ND						
BY BY	NC ND						
BY BY	NC ND						
© (I)	NC ND						
BY BY	NC ND						
BY BY	NC ND						
BY	NC ND						