

Insertion sort

1. Introduction
2. Algorithm presentation
3. Operation counts: best, worst, average

1. Introduction

Insertion sort is a sorting algorithm.

Sorting: placing elements in order (smallest to largest or largest to smallest)

Assumption: for two elements x , y , only three possible cases: (i) $x < y$, (ii) $x = y$ or (iii) $x > y$

Insertion-sort:

- can be used to sort items/elements in an array or list
- an example of decrease and conquer
- easy to understand/code
- one of many different sorting algorithms

2. Algorithm presentation

Description level

1. We start with array $A[1..n]$ of numbers in no particular order. We want rearrange the array so that the numbers are in order from smallest to largest.
 2. We handle one number at a time. Let $A[j]$ be number we are handling. All numbers before $A[j]$ are sorted. When handling $A[j]$, we look for correct position k , $1 \leq k \leq j$ where $A[j]$ belongs. When we find k , we shift all elements in $A[k..j-1]$ right one position and put $A[j]$ into position k .
 3. After handling $A[j]$, we handle $A[j+1]$. We repeat until all numbers have been handled.
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Example

	start				
i	1	2	3	4	5
A[i]	24	17	-6	24	-3

	goal				
i	1	2	3	4	5
A[i]	-6	-3	17	24	24

when handling element A[5]

A[i]	-6	-3	17	24	24
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↑



Pseudocode

```
1  INSERTSORT(A)
2  input: number array A output: sorted array A
3  /* The numbers in input A[1..n] may be in any order. On output the
4  numbers in A are sorted from smallest to largest. */
5  for j from 2 to A.length
6      key = A[j], k = j
7      while k ≥ 2 and A[k - 1] > key
8          A[k] = A[k - 1], k = k - 1
9      end
10     A[k] = key
11 end
```

Example

j	key	A[1..n] before line 6	A[1..n] after line 10										
1	—	<table border="1"><tr><td>24</td><td>17</td><td>-6</td><td>24</td><td>-3</td></tr></table>	24	17	-6	24	-3	—					
24	17	-6	24	-3									
2	17	<table border="1"> <tr><td>24</td><td>17</td><td>-6</td><td>24</td><td>-3</td></tr> </table> <div style="text-align: center;"> ↘ ↑ ↑ > 17? </div>	24	17	-6	24	-3	<table border="1"><tr><td>17</td><td>24</td><td>-6</td><td>24</td><td>-3</td></tr></table>	17	24	-6	24	-3
24	17	-6	24	-3									
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3	-6	<table border="1"> <tr><td>17</td><td>24</td><td>-6</td><td>24</td><td>-3</td></tr> </table> <div style="text-align: center;"> ↘ ↘ ↑ ↑ > -6? </div>	17	24	-6	24	-3	<table border="1"><tr><td>-6</td><td>17</td><td>24</td><td>24</td><td>-3</td></tr></table>	-6	17	24	24	-3
17	24	-6	24	-3									
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□

3. Operation counts: best, worst, average

Pseudocode

```

1  INSERTSORT(A)
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```

How many times will each line be executed?

- for-loop goes through $A.length - 1$ elements; line 6 (or 5 or 10) executed $A.length - 1$ times $(n-1)$
- how many times line 8 executed depends on initial order of $A[1..n]$

Best

Is it possible we never need to execute line 8? What conditions?

Yes.

If array was already sorted at start.

Worst

Consider $j = n$ (element $A[n]$). Most number of times line 8 must be executed? $(n-1)$

Consider $j = n-1$ (element $A[n-1]$). Most number of times line 8 must be executed? $(n-2)$

$A[1]$	$A[2]$	$A[3]$	\dots	$A[n-1]$	$A[n]$
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Average

Consider $j = n$ (element $A[n]$). Best assumption on how many numbers in $A[1..(n-1)]$ are greater than $A[n]$? $(n-1)/2$

Consider $j = n-1$ (element $A[n-1]$). Best assumption on how many numbers in $A[1..(n-2)]$ are greater than $A[n-1]$? $(n-2)/2$

j

most number of times
line 8 executed

average number of times
line 8 executed

n

$(n-1)$

$(n-1) / 2$

$n-1$

$(n-2)$

$(n-2) / 2$

\vdots

i

$(i-1)$

$(i-1) / 2$

\vdots

2

1

$1 / 2$

$$\sum_{r=1}^{n-1} r = \frac{n(n-1)}{2}$$

$$\frac{1}{2} \sum_{r=1}^{n-1} r = \frac{n(n-1)}{4}$$

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